Part 11: Fluids

Physics for Engineers & Scientists (Giancoli): Chapter 10
University Physics V1 (Openstax): Chapters 14

Fluids (Static)

- Unlike rigid objects, fluids flow. Often they don't have a fixed shape and sometimes not even a fixed size.
- Rather than using masses and forces, it is more convenient to use mass density (ρ), often referred to as just 'density', and pressure (P).
- The Density of a substance is its mass divided by volume. Units of Density: kg/m^3.

$$\rho = \frac{m}{V}$$

- The Specific Gravity of a substance is the ratio of its density to the density of water at 4°C.

$$\rho_{H_2O(4°C)} = 1000\frac{kg}{m^3}$$

- The molecules in a fluid are in random motion and collide with whatever contains them. The impulse of these collisions impart forces (simultaneously holding the fluid inside and pressing outward on the container). This force per area is defined to be The Pressure (P) of the fluid.

$$P = \frac{F}{A}$$

  - The Units of Pressure are the Pascal. $1\ Pa\ = \frac{N}{m^2}$.
  - Another common unit is the 'atmosphere' (atm), which is roughly the pressure of the air at Earth's surface.

$$1\ atm = 1.013\ \times\ 10^5\ Pa = 101.3\ kPa$$

- Forces from pressure are always directed perpendicular (normal) to the surface.

Example: The pressure inside a plastic bottle containing a carbonated beverage is 207 kPa. The diameter of the cap is 2.22 cm. Determine the net force of pressure on the cap.

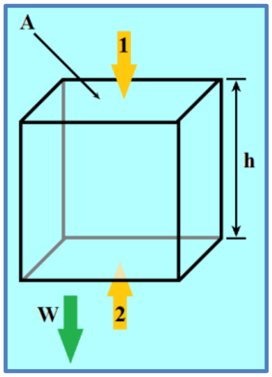
The net force is the force from the inside pressure minus the force from the outside pressure.

$$A = \pi r^2 = \pi\left(\frac{d}{2}\right)^2 = \pi\left(\frac{0.0222\ m}{2}\right)^2 = 3.871\ \times\ 10^{-4}\ m^2$$

$$F_{Net} = F_{Inside} - F_{Outside} = P_{Inside}\cdot A - P_{Outside}\cdot A = (P_{Inside} - P_{Outside})\cdot A$$

$$F_{Net} = [(2.07\ \times\ 10^5\ Pa) - (1.013\ \times\ 10^5\ Pa)]\cdot(3.871\ \times\ 10^{-4}\ m^2) = 40.9\ N$$

Pressure with Depth (Static): $P_2 = P_1 + \rho g h$



Let's take a large mass of a static fluid. We can place an imaginary container (a box) around some of the fluid. Let's examine this object.

The forces on the sides cancel (left/right, front/back) otherwise there would be horizontal acceleration of the fluid enclosed.

As our fluid is stationary, the vertical pressures at top and bottom must cancel out the weight of the fluid enclosed. $F_2 = F_1 + W$

$$P_2 A = P_1 A + mg = P_1 A + \rho V g = P_1 A + \rho A h g \qquad P_2 = P_1 + \rho g h$$

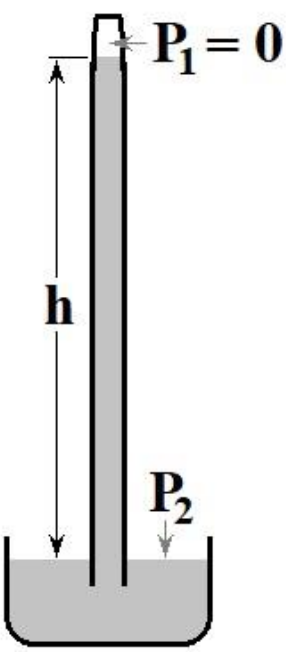
Example: The deepest point in Earth's oceans is in the Mariana Trench at a depth of 10,994 m. If we were to explore this region in a bathysphere, what would be the net force on a 30.0 cm diameter window? You may assume the density of seawater is 1030 kg/m^3 and that the pressure inside the bathysphere is kept at 1.00 atm.

$$F_{Net} = F_{Out} - F_{In} = P_{Out}A - P_{In}A = (P_{Out} - P_{In}) \cdot A = [(P_{atm} + \rho g h) - P_{atm}]\pi r^2$$

$$F_{Net} = \pi \rho g h r^2 = \pi \left(1030\,\frac{kg}{m^3}\right)\left(9.80\,\frac{m}{s^2}\right)(10{,}994\ m)(0.150\ m)^2 = 7.84\ \times 10^6\ N$$

7.84 × 10^6 N is roughly equivalent to 1.7 million lbs.

Pressure Gauges



- A simple barometer (pressure gauge) can be built by filling a tube with fluid and then turning it over (without allowing air to enter) into a reservoir.
- The difference in pressure between the outside air pressure and the vacuum at the top is sufficient to support the column of fluid of height h.
- The height of the column of fluid is directly proportional to the pressure.

$$P_2 = P_1 + \rho g h = \rho g h \qquad h = \frac{P_2}{\rho g}$$

- To keep the height of the column to a minimum a high density liquid, such as mercury (Hg) must be used.

$$\rho_{Hg} = 13.6 \times 10^3\,\frac{kg}{m^3} \qquad h = \frac{P_2}{\rho g} = \frac{\left(1.013 \times 10^5\ Pa\right)}{\left(13.6{\times}10^3\frac{kg}{m^3}\right)\left(9.80\frac{m}{s^2}\right)} = 760\ mm$$

The units 'mmHg' (millimeters of mercury) are often used in weather.

Gauge Pressure

- Some types of gauges (such as those that measure the air pressure in tires) only measure the difference between the pressure measured and atmospheric pressure.
    - The measured value is called the **gauge pressure**.
    - The actual pressure is called the **absolute pressure**.

$$P_{Absolute} = P_{Gauge} + P_{atm}$$

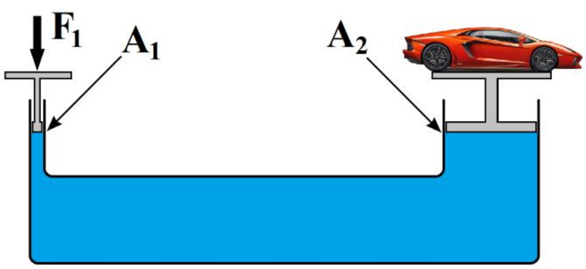
The barometer with the column of fluid supported by pressure measures the absolute pressure.

The pressure with depth equation is valid with both absolute and gauge pressures. However, most equations are only valid with absolute pressure.

Pascal's Principle

"Any change in pressure applied to a completely enclosed fluid is transmitted undiminished to every part of the fluid and to the walls enclosing it."

This is a direct result of $P_2 = P_1 + \rho g h$.
Any change to P_1 changes P_2 as well, and P_2 could be anywhere in the fluid.



The two pistons are at the same height. They must have the same pressure.

$$P_1 = P_{atm} + \frac{F_1}{A_1} \qquad P_2 = P_{atm} + \frac{F_2}{A_2}$$

$$\frac{F_1}{A_1} = \frac{F_2}{A_2} \qquad F_1 = \frac{A_1}{A_2} F_2$$

The mechanical advantage created in this way is the basis of hydraulics.

Example: The input piston of a hydraulic lift in an automotive repair shop has a radius of 1.00 cm. The output piston has a radius of 32.0 cm. What force is needed at the input piston to lift the 1570 kg car on the output piston?

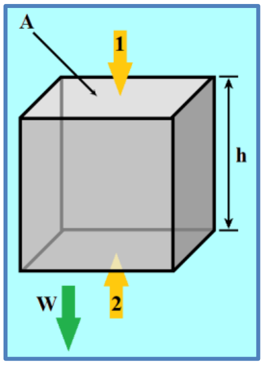
$$F_1 = \frac{A_1}{A_2} F_2 = \frac{\pi r_1^2}{\pi r_2^2} F_2 = \left(\frac{r_1}{r_2}\right)^2 F_2 = \left(\frac{r_1}{r_2}\right)^2 mg = \left(\frac{1.00\ cm}{32.0\ cm}\right)^2 (1570\ kg)\left(9.80 \frac{m}{s^2}\right) = 15.0\ N$$

Energy is still conserved. For every millimeter the car moves, the input piston must move a meter.

Archimedes' Principle

"The buoyant force is equal to the weight of the fluid displaced."

- The net sum of the force of pressure on all sides of an object is called the Buoyant Force, and it points upward.



Let's take a large mass of a static fluid and place and object inside. We can calculate the buoyant force on this object.

$$F_B = F_2 - F_1 = P_2A - P_1A = [P_2 - P_1]A =$$

$$F_B = [(P_1 + \rho_F gh) - P_1]A = \rho_F ghA = \rho_F gV =$$

$$F_B = m_{DF}g = W_{DF}$$

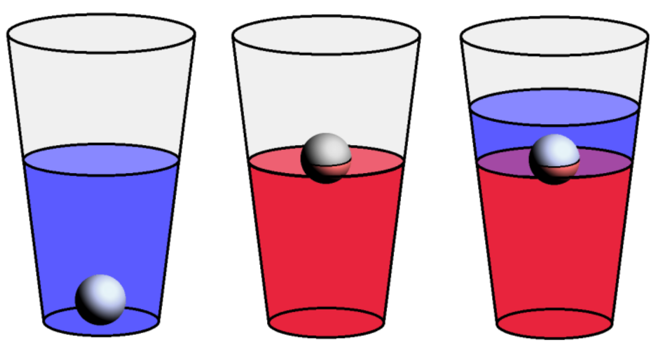
Archimedes' Principle: The buoyant force on an object is equal to the weight of the fluid displaced.

If the object is replaced with fluid (the same fluid), the fluid filling the boundaries is static. If the fluid is static, then the net force must be zero. If the net force is zero, then the buoyant force must equal to the weight. The buoyant force doesn't change when we return the original object.

- To get the Apparent Weight (W_{App}) of an object subtract the buoyant force from the weight.

Objects in water are easier to lift as the buoyant force helps us.

Conceptual Example: If blue fluid is added to the glass with red fluid, will the ball rise or fall?



The grey ball sinks to the bottom when it is placed in the glass with the blue fluid.

The grey ball floats with half of its volume submerged when placed in the glass with the red fluid.

If the blue fluid is slowly added to the top of the red fluid (with the ball floating on top) will the ball rise or fall?

Before the blue fluid is added, the ball has half of its volume submerged.

$$F_B = W_{DF} = \rho_{RF}\left(\frac{1}{2}V_{Ball}\right)g + \rho_{Air}\left(\frac{1}{2}V_{Ball}\right)g = \left(\frac{1}{2}\rho_{RF} + \frac{1}{2}\rho_{Air}\right)V_{Ball}\cdot g = W_{Ball}$$

If the ball remains in the same place (when the blue fluid is added), here is the buoyant force:

$$F_B = W_{DF} = \rho_{RF}\left(\frac{1}{2}V_{Ball}\right)g + \rho_{BF}\left(\frac{1}{2}V_{Ball}\right)g = \left(\frac{1}{2}\rho_{RF} + \frac{1}{2}\rho_{BF}\right)V_{Ball}\cdot g$$

As $\rho_{BF} > \rho_{Air}$, then: $\left(\frac{1}{2}\rho_{RF} + \frac{1}{2}\rho_{BF}\right)V_{Ball}\cdot g > \left(\frac{1}{2}\rho_{RF} + \frac{1}{2}\rho_{Air}\right)V_{Ball}\cdot g = W_{Ball}$

As the buoyant force is greater than the mass of the ball, the ball will rise until the buoyant force is equal to the weight.

<u>Example</u>: A duck is floating on the water with half of its volume underwater. Determine the density of the duck.

$$F_B = W_{Duck} = m_{Duck} \cdot g = \rho_{Duck} \cdot V_{Duck} \cdot g$$

$$F_B = W_{DF} = m_{DF} \cdot g = \rho_{DF} \cdot V_{DF} \cdot g = \rho_{DF} \cdot \left(\frac{1}{2} V_{Duck}\right) \cdot g = \frac{1}{2}\rho_{DF} \cdot V_{Duck} \cdot g$$

$$\rho_{Duck} \cdot V_{Duck} \cdot g = \frac{1}{2}\rho_{DF} \cdot V_{Duck} \cdot g$$

$$\rho_{Duck} = \frac{1}{2}\rho_{DF} = \frac{1}{2}\left(1000\frac{kg}{m^3}\right) = 500\frac{kg}{m^3}$$

Fluids in Motion

- In **Steady Flow**, all the particles are moving at the same speed as they pass a given point.
- In **Unsteady Flow**, particles are moving at different speeds as they pass a given point.
- In **Turbulent Flow**, velocities can change radically at a given point.
- A fluid is **Compressible** if the density of the fluid changes with pressure.
- A fluid is **Incompressible** if the density of the fluid is constant with changes with pressure.

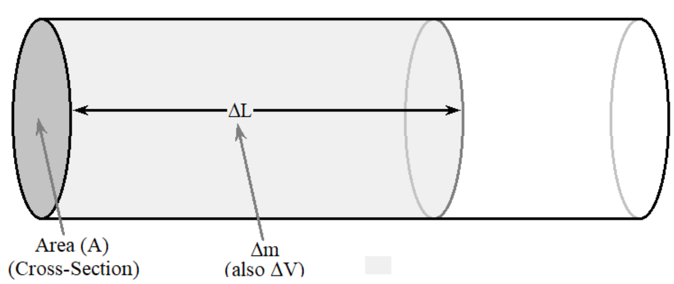
Fluids are nearly incompressible/ Gases are compressible.

- A **Viscous** fluid doesn't flow readily.
- A **Non-viscous** fluid flows readily.

We will assume that all fluids are Ideal Fluids, meaning they are incompressible and non-viscous. We will also assume that the flow is steady.

Mass Flow Rate

- **Mass Flow Rate** is the amount of mass that passes through a cross-section of the pipe in a given time interval.



$$MFR = \frac{\Delta m}{\Delta t} = \frac{\rho \Delta V}{\Delta t} = \frac{\rho A \Delta L}{\Delta t} = \rho A v$$

ΔV is the volume occupied by the mass that will move through the cross section. ΔL is the length of ΔV, and if this masses through the cross-section in Δt, then the velocity of the fluid must be ΔL/ Δt.

- Contained fluid (such as in a pipe) has a fixed volume. If we assume it is an ideal fluid with a constant density, then it also has a fixed mass. Any mass that enters one end implies that an equal mass leave the other end. In other words, the mass flow rate is constant.

$$\rho A_1 v_1 = \rho A_2 v_2$$

- Dividing out the density gives the **Volume Flow Rate**, which is also constant.

$$A_1 v_1 = A_2 v_2$$

Example: Water flows through a pipe of diameter 0.500 m at a velocity 0.250 m/s. The water flow is then constricted to a pipe of diameter 0.250 m. Determine A) the mass flow rate, and B) the velocity in the narrow pipe.

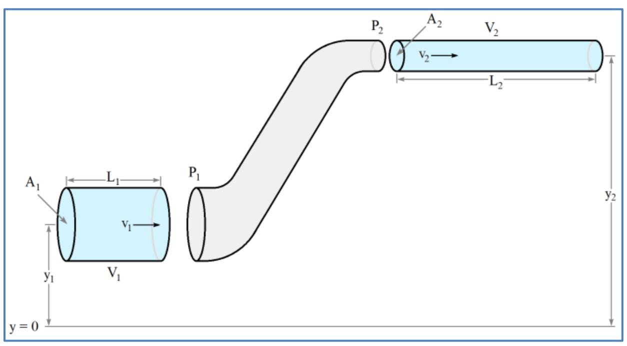
$$MFR = \rho A v = \left(1000 \frac{kg}{m^3}\right) \pi \left(\frac{0.500\ m}{2}\right)^2 \left(0.250 \frac{m}{s}\right) = 49.1 \frac{kg}{s}$$

$$A_1 v_1 = A_2 v_2$$

$$v_2 = \frac{A_1}{A_2} v_1 = \frac{\pi r_1^2}{\pi r_2^2} v_1 = \left(\frac{r_1}{r_2}\right)^2 v_1 = \left(\frac{d_1}{d_2}\right)^2 v_1 = \left(\frac{0.500\ \text{m}}{0.250\ \text{m}}\right)^2 \left(0.250 \frac{\text{m}}{\text{s}}\right) = 1.00 \frac{m}{s}$$

The Bernoulli Equation

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$



If we push water into the pipe, an equal amount of water must come out the other side. (V_1=V_2)

If there is steady flow through the pipe, then energy must be conserved in this process.

The initial and final states have kinetic and gravitational potential energy, but energy is also added by work done by pressure.

$$E_{Init} = \frac{1}{2} m v_1^2 + m g y_1 = \frac{1}{2}\rho V_1 v_1^2 + \rho V_1 g y_1$$

$$E_{Final} = \frac{1}{2} m v_2^2 + m g y_2 = \frac{1}{2}\rho V_2 v_2^2 + \rho V_2 g y_2$$

$$E_{added} = W_1 + W_2 = F_1 L_1 - F_2 L_2 = P_1 A_1 L_1 - P_2 A_2 L_2 = P_1 V_1 - P_2 V_2$$

$$E_{Init} + E_{added} = E_{Final}$$

$$\frac{1}{2}\rho V_1 v_1^2 + \rho V_1 g y_1 + P_1 V_1 - P_2 V_2 = \frac{1}{2}\rho V_2 v_2^2 + \rho V_2 g y_2$$

$$\frac{1}{2}\rho v_1^2 + \rho g y_1 + P_1 - P_2 = \frac{1}{2}\rho v_2^2 + \rho g y_2$$

$$\boldsymbol{P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2}$$

Example: During a tornado the winds and pressure can be sufficient to rip the roofs off houses. The roof of a cabin is 10.0 m by 7.00 m. The air outside is moving at 67.0 m/s (roughly 150 mph). You may assume that the difference in height from the top of the roof to the bottom is negligible. The density of air is 1.225 kg/m^3. How much force is exerted on the roof?

$$F_{Net} = F_{inside} - F_{outside} = P_{inside}A - P_{outside}A = (P_{inside} - P_{outside})A = (P_1 - P_2)LW$$

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

As stated in the problem, $y_1 \approx y_2$ (ρgy terms cancel). Also, the air is still inside ($v_1 = 0$).

$$P_1 = P_2 + \frac{1}{2}\rho v_2^2$$

$$P_1 - P_2 = \frac{1}{2}\rho v_2^2 = \frac{1}{2}\left(1.225\frac{kg}{m^3}\right)\left(67.0\frac{m}{s}\right)^2 = 2{,}749.5\ Pa$$

$$F_{Net} = (P_1 - P_2)A = (2{,}749.5\ Pa)(10.0\ m)(7.00m) = 192\ kN$$

192 kN is roughly equal to 43,000 lbs.

Example: The top was removed from an old silo to convert it to collecting and storing rain water. The silo is 10.0 m high, 2.00 m in radius, and full to the open top with water. A poorly placed shot from a rifle puts a hole in the side of the silo that is 1.00 cm in diameter and 1.60 m above the ground. Determine the velocity of the water as it streams out the hole.

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

P_1 and P_2 are both 1 atm (open to the air) and will cancel.

$$\tfrac{1}{2}\rho v_1^2 + \rho g y_1 = \tfrac{1}{2}\rho v_2^2 + \rho g y_2 \qquad \rho g y_1 - \rho g y_2 = \tfrac{1}{2}\rho v_2^2 - \tfrac{1}{2}\rho v_1^2 \qquad \rho g(y_1 - y_2) = \tfrac{1}{2}\rho(v_2^2 - v_1^2)$$

Need to get v_1 (top of the silo) in terms of v_2 (coming out the bullet hole).

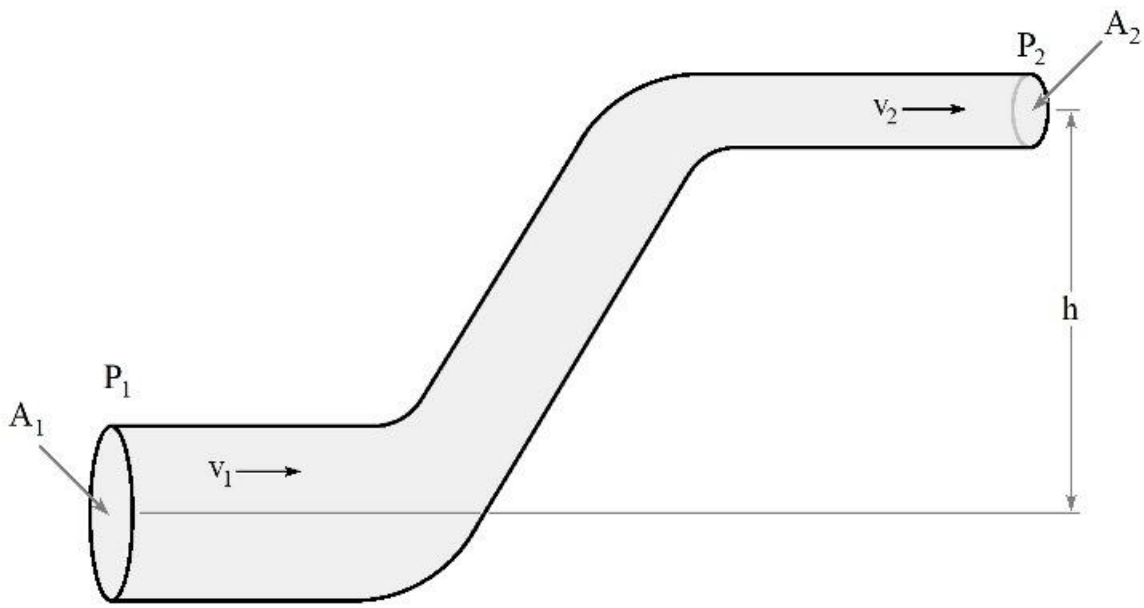
$$A_1 v_1 = A_2 v_2 \qquad v_1 = \frac{A_2}{A_1}v_2 = \frac{\pi r_2^2}{\pi r_1^2}v_2 = \left(\frac{r_2}{r_1}\right)^2 v_2 = \left(\frac{0.005\ m}{2.00\ m}\right)^2 v_2 = (6.25\times 10^{-6})\ v_2$$

As v_1 is roughly a million times smaller than v_2, the v_1^2 is negligible compared to v_2^2.
Keep this in mind. In many cases, one of the KE terms is negligible.

$$\rho g(y_1 - y_2) = \tfrac{1}{2}\rho v_2^2 \qquad v_2^2 = 2g(y_1 - y_2)$$

$$v_2 = \sqrt{2g(y_1 - y_2)} = \sqrt{2\left(9.80\frac{m}{s^2}\right)(10.0\ m - 1.60\ m)} = 12.8\frac{m}{s}$$

Example: In order to clear an obstacle, an oil pipeline rises h = 12.5 m in height. As it does the pipeline narrows from a diameter of 1.20 m to a diameter of 0.625 m. The density of the crude oil is 827 kg/m^3, and the velocity of the oil at the bottom is v_1 = 1.25 m/s. If the pressure at the top (P_2), can't exceed 56.7 atm, determine the maximum pressure of P_1.



$$A_1 v_1 = A_2 v_2 \qquad v_2 = \frac{A_1}{A_2} v_1 = \frac{\pi r_1^2}{\pi r_2^2} v_1 = \left(\frac{r_1}{r_2}\right)^2 v_1 = \left(\frac{d_1}{d_2}\right)^2 v_1 = \left(\frac{1.20\ m}{0.625\ m}\right)^2 \left(1.25 \frac{m}{s}\right) = 4.608 \frac{m}{s}$$

$$P_2 = 56.7\ atm \left(\frac{1.013 \times 10^5\ Pa}{1\ atm}\right) = 5.74371 \times 10^6\ Pa$$

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

$$P_1 = P_2 + \frac{1}{2}\rho(v_2^2 - v_1^2) + \rho g(y_2 - y_1) = P_2 + \frac{1}{2}\rho(v_2^2 - v_1^2) + \rho g h$$

$$P_1 = 5.74371 \times 10^6\ Pa + \frac{1}{2}\left(827 \frac{\text{kg}}{\text{m}^3}\right)\left[\left(4.608 \frac{m}{s}\right)^2 - \left(1.25 \frac{m}{s}\right)^2\right] + \left(827 \frac{\text{kg}}{\text{m}^3}\right)\left(9.80 \frac{\text{m}}{\text{s}^2}\right)(12.5\text{m})$$

$$P_1 = 5.74371 \times 10^6\ Pa + 8134\ Pa + 101{,}308\ Pa = 5.85 \times 10^6\ Pa$$